Department of Mathematics & Philosophy of Engineering
Faculty of Engineering Technology
The Open University of Sri Lanka –Nawala, Nuggegoda
Nawala - Nugegoda

Course: MPZ 3132-Engineering Mathematics IB Model Answer No.01 Academic Year 2011/2012

(1)

1.1.1

Consider a annulus of radius r and with r. The area of this annually = $2\pi r dr$

Let ρ he the density of the body

$\therefore$ the mass of this annually = $2\pi r\rho dr$

$\therefore$ The moment of inertia of the above annulus about an ----- 0 perpendicular to the plane of the lamina = $2\pi r^3\rho dr$

$\therefore$ The moment of inertia of the whole annulus = $\int_a^b e\pi r^3\rho dr$

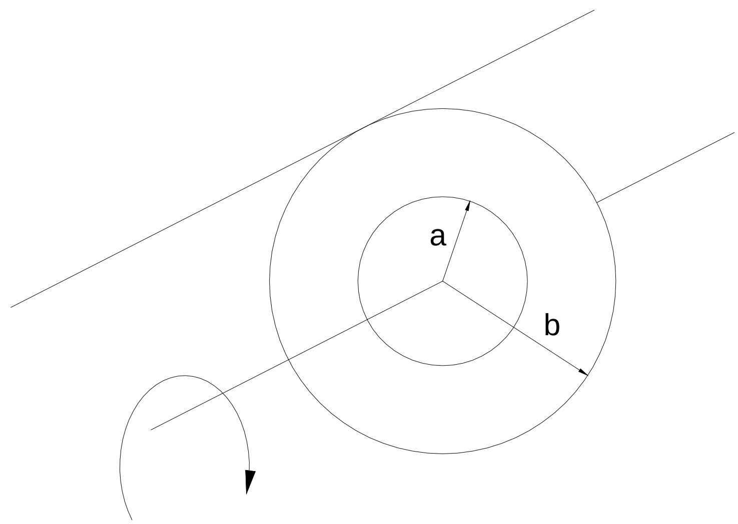
$$I = 2\pi\rho\left[\frac{r^4}{4}\right]_a^b$$

$$I = 2\pi\rho\frac{\left(b^4 - a^4\right)}{4}$$

$$\rho = \frac{\mu}{\pi\left(b^2 - a^2\right)}$$

$$I = \frac{2\pi\rho l\left(b^4 - a^a\right)}{\pi\left(b^2 - a^2\right)}$$

$$I = \frac{1}{2}M\left(b^2 + a^2\right)$$

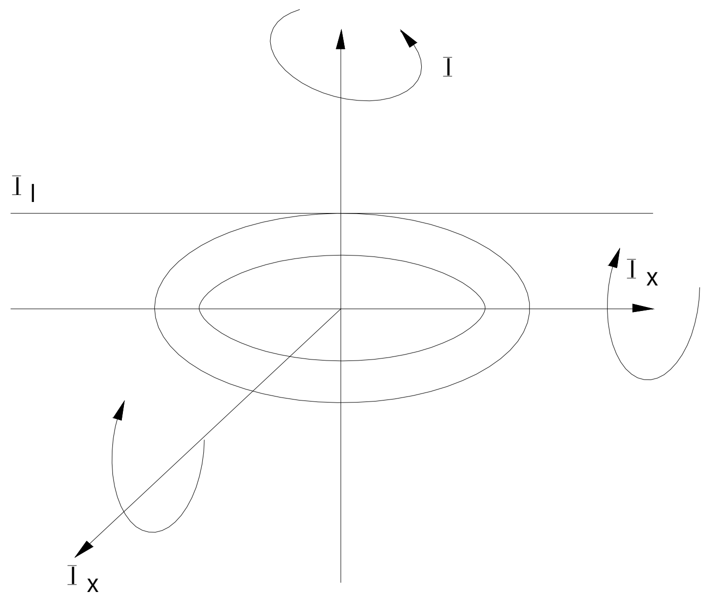


1.1.2

$\therefore$ Using perpendicular axis theorem

$2I_x = I$

$$I_x = \frac{1}{4}\left(b^2 + a^2\right)M$$



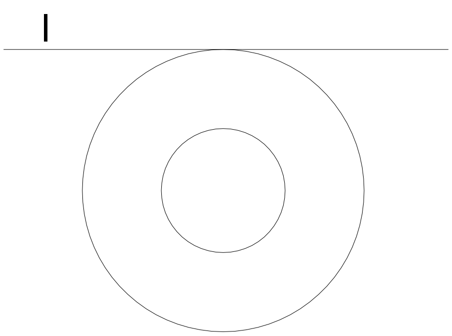
Using parallel axis theorem

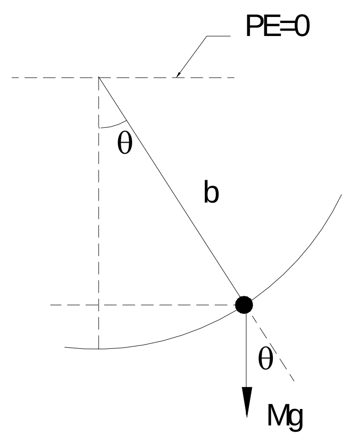
$$I_i = Mb^2 + I_x$$

$$= Mb^2 + \frac{1}{4}M(a^2 + b^2)M$$

$$= \frac{1}{4}\left(a^2 + 5b^2\right)M$$

side view





Applying energy equation

$$\frac{1}{2}I_e\theta^2 = \text{Mgbone} = \text{constant}$$

Differencing with respect to time

$$\frac{1}{2}\;\; T_l 2\dot{\theta}\;\ddot{\theta} + \text{Mgbsin}\theta\;\ddot{\theta} = 0$$

$$\ddot{\theta} = -\frac{Mgh}{I_l}\sin\theta$$

When θ is small $\sin\theta = \theta$

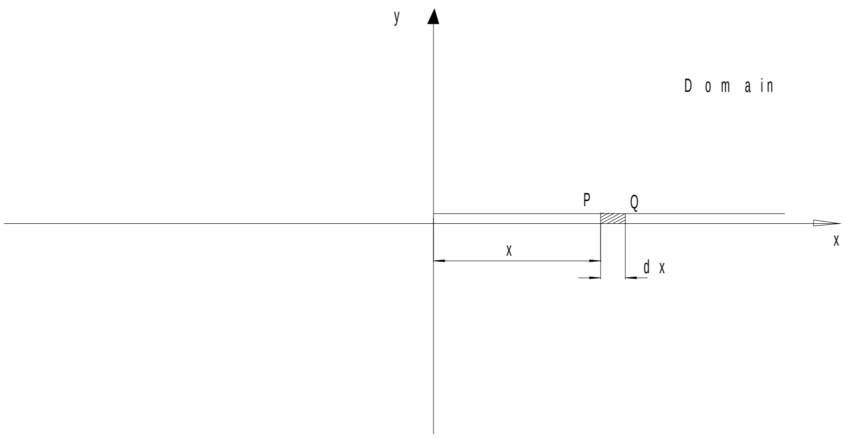
$$\ddot{\theta} = -\frac{Mgh}{\frac{1}{4}\left(9^2 + 56^2\right)M}\theta$$

$$\ddot{\theta} = -\frac{4gh}{a^2 + 5b^2}\theta$$

$$w = 2\sqrt{\frac{gh}{a^2 + 5b^2}}$$

$$T = \frac{2\pi}{w} = \pi\sqrt{\frac{a^2 + 5b^2}{gb}}$$

(2)



$M = 2a\sigma$

$\sigma = \dfrac{M}{2a}$

σ = linear density

mass of PQ = σdx

The moment of inertia of PQ about I_x

$= \pi^2 \rho dx$

$\therefore$ The moment of inertia of the rod about y – axis = $\displaystyle\int_{-0}^{2a} x^2 \rho dx$

$$= \rho\left[\frac{x^3}{3}\right]_0^{2a}$$

$$= \rho\frac{3a^3}{3}$$

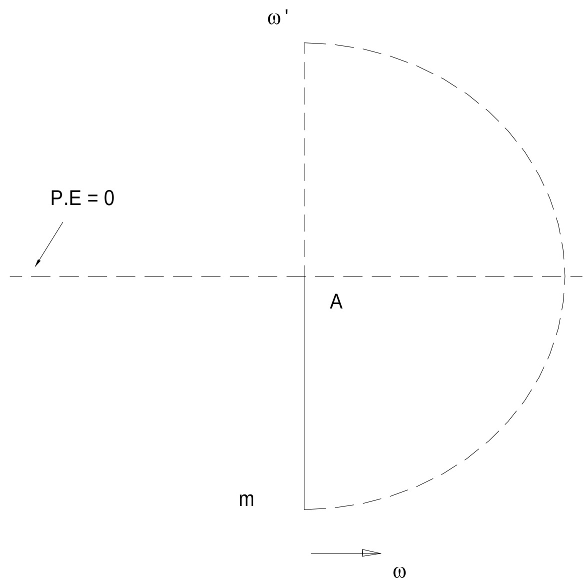
$$= \frac{M}{2a}\frac{8}{3}a^3 = \frac{4}{3}Ma^2$$

2.1
2.1.1

$$I_A = \frac{4}{3}Ma^2 + m4a^2 I$$

$$= \frac{4a^2}{3}\left(M + 3m\right)$$

2.1.2
Using energy equation law



$$\frac{1}{\partial} I_a w^2 - mg.2a - Mga$$

$$= \frac{1}{\partial} I_\theta \ w_1^2 + Mga +, mg2a$$

$$\frac{1}{2} I_A w_1^2 = \frac{1}{z} I_A w^2 - 4mga - 2Mga$$

To describe complete circles

$$\frac{1}{2} I_A w_1^2 \geq 0$$

$$\frac{1}{2} I_A w^2 \geq 2ag(M + 2m)$$

$$w^2 \geq \frac{2dg\left(M + 2m\right)}{\frac{4a^x}{3}\left(M + 3m\right)}$$

$$w \geq \sqrt{\frac{33\left(M + 2m\right)}{29\left(M + 3m\right)}}$$

(3)

Just before the particles ------ with body the momentum of the body = m $\underline{\upsilon}$
The momentum of the practically = $\delta m\underline{u}$
Immediately afterwards
The momentum of the augmented body = $\left(m + \delta m\right)\left(\underline{v} + \delta\underline{v}\right)$
The increase momentums of the system in time $\delta t = \left(m + \delta m\right)\left(\underline{\upsilon} + \delta\underline{v}\right) - m\underline{\upsilon} - m\delta\underline{\upsilon}$
Impulse of the force acting on the body in same sine interval = $\underline{F}\delta$ t

$\therefore \underline{F}\,\delta t = (m+\delta m)(\underline{\upsilon}+\delta\underline{v}) - m\underline{\upsilon} - \underline{u}\delta m$

$\underline{F} = m\dfrac{\delta \underline{v}}{\delta t} + \underline{\upsilon}\dfrac{\delta m}{\delta t} + \delta m\dfrac{\delta \underline{\upsilon}}{\delta t}$

As $\delta t \to 0$ $\qquad \delta \underline{\upsilon} \to c$

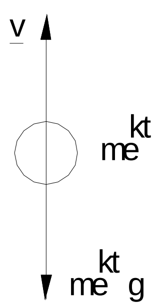
$\underline{F} = m\dfrac{dv}{dt} + \underline{\upsilon}\dfrac{dm}{dt} + 0.\dfrac{d\underline{\upsilon}}{dt} - \underline{u}\dfrac{dm}{dt}$

$\underline{F} = m\dfrac{d\underline{\upsilon}}{dt} + (\underline{\upsilon}-\underline{u})\dfrac{dm}{dt}$

$\underline{F} = m\dfrac{d\underline{\upsilon}}{dt} + (\underline{\upsilon}-\underline{u})\dfrac{dm}{dt}$

3.1

If $u = 0$ $\qquad \underline{F} = \dfrac{d}{dt}(m\underline{v})$



Applying $\uparrow$ $\underline{\underline{F}} = m\dfrac{d\underline{\upsilon}}{dt} + (\underline{v}-\underline{u})\dfrac{dm}{dt}$

Since $\underline{u} = \underline{c}$

$\underline{F} = \dfrac{d}{dt}(m\underline{v})$

$-m\,e^{ht}g = \dfrac{d}{dt}\left(m\underline{\upsilon}\,e^{ht}\right)$

Initial momentum $m\dfrac{g}{h}$

If $t = T$ at the highest point

$-\int_0^T mge^{ht}dt = \left[m\upsilon e^{ht}\right]_{\frac{mg}{h}}^{0}$

$-mg\left[\dfrac{e^{ht}}{k}\right]_0^T = \left[m\upsilon e^{ht}\right]_{mg}^{0}$

$$\frac{mg}{h}\left(e^{h}+1\right)=0-m\frac{g}{h}$$

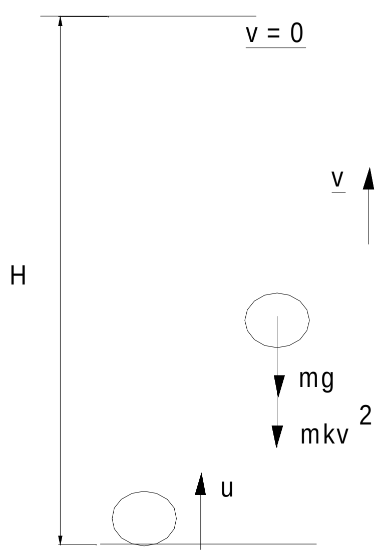
$$\frac{me^{ht}}{h}=\frac{2m}{h}$$

$me^{ht} = 2m$

$\therefore$ The mass of the body at the highest point is 2m

Applying $\underline{F}=m\underline{a}$

$$-(ny + mh\upsilon^2) = mv\frac{dv}{dx}$$



$$dx=\frac{-v}{g+hv^2}d\upsilon$$

$$x=-\int \frac{\upsilon}{g+hv^2}d\upsilon+\text{const}$$

$$x=-\frac{1}{2h}\lg(g+hv^2)+c$$

$v = u,\ \ x = 0$

$$c=+\frac{1}{2h}\log\left(g+hu^2\right)$$

$$\therefore x=\frac{1}{2h}\log\left(g+hu^2\right)-\frac{1}{2h}\lg(g+hv^2)$$

$$x=\frac{1}{2h}\log\left|\frac{g+hu^2}{g+hv^2}\right|$$

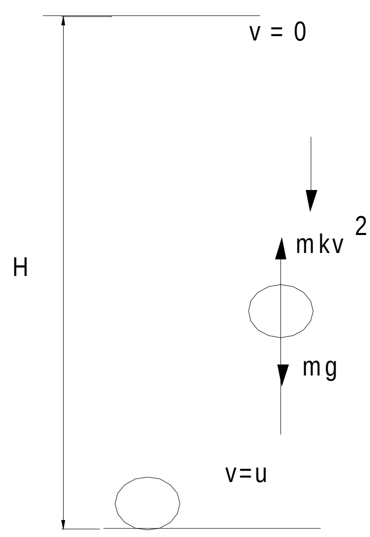
$$x_{max}=\frac{1}{2h}\log\left[\frac{g+hu^2}{g}\right]$$

$$H=\frac{1}{\partial h}\ln\left|1+\frac{hu^2}{g}\right|$$

3.2

Applying $\underline{F}=m\underline{a}$

$$Mg - mhv^2 = m\upsilon\frac{dv}{dm}$$



$$dx=\frac{\nu}{g-hv^2}dv$$

$$x=\frac{-1lm}{2h}\left(g-hv^2\right)+c'$$

$c' = \text{constant}$

at x = 0, v = 0

$$\therefore \quad c' = \frac{1}{2h} \ln g$$

$$\therefore \quad x = \frac{1}{2h}\left[\text{lug} - \text{lu}\left(g - hv^2\right)\right]$$

$$x = \frac{1}{2h} \text{lu}\left[\frac{3}{g - hv^2}\right]$$

V = α u, x = H

$$\frac{1}{2h} \text{lu}\left|\frac{g}{g - hd^2u^2}\right| = \frac{1}{2u} \text{lu}\left|\frac{g + hu^2}{g}\right|$$

$g^2 = (g + hu^2)(g - h\,\alpha^2 u^2)$

$$g^2 = g^2 + hgu^2 - hg\alpha^2u^2 - h^2\alpha^2u^*$$

$O = g - g\alpha^2 - \alpha^2u^2h$

$\alpha^2 (g + u^2k) = 9$

$$\frac{\left(g + u^2h\right)}{g} = \frac{1}{\alpha^2}$$

$$1 + \frac{u^2h}{g} = \frac{1}{\alpha^2}$$

(4)

4.1

$$OP = \underline{r} = v\cos\theta\underline{i} + v\sin\theta\underline{z}$$

$\therefore$ Unit vector along OP = $\dfrac{|r}{|\underline{r}|}$

$$\therefore \quad |\underline{r}| = r$$

$$\therefore \underline{i} = \cos\theta\underline{i} + \sin\theta\underline{j}$$

$$NM = -r\sin\theta\tan\theta\underline{i} = -r\frac{\sin^2\theta}{\cos\theta}\underline{i}$$

$$MP = r\sin\theta\underline{j}$$

$$\therefore NP = NM + MP = -r\frac{\sin^2\theta}{\cos\theta} i + r\sin\theta\underline{j}$$

$$|NP| = r\tan\theta$$

$$\therefore m = -\sin\theta\underline{i} + \cos\theta\underline{j}$$

$$\mathrm{i} = \frac{\mathrm{dl}}{\mathrm{dt}} = \frac{\mathrm{d}}{\mathrm{dt}}\left(\cos\theta\underline{\mathrm{i}} + \sin\theta\underline{\mathrm{j}}\right) = -\sin\theta\dot{\theta}\underline{\mathrm{i}} + \cos\theta\dot{\theta}\underline{\mathrm{j}}$$
$$\underline{\mathrm{i}} = \left(-\sin\theta\underline{\mathrm{i}} + \cos\theta\underline{\mathrm{j}}\right)\dot{\theta}$$
$$\underline{\mathrm{i}} = \underline{\mathrm{m}}\dot{\theta}$$
$$\underline{\mathrm{m}} = -\cos\dot{\theta}\underline{\mathrm{i}} - \sin\theta\dot{\theta}\underline{\mathrm{j}} = -\dot{\theta}\left(\cos\theta\underline{\mathrm{i}} + \sin\theta\underline{\mathrm{j}}\right)$$
$$\underline{\mathrm{m}} = -\theta\underline{\mathrm{i}}$$

$\therefore$ The position vector of P is $\underline{\mathrm{r}}$

$$\underline{\mathrm{r}} = \mathrm{r}\cos\underline{\mathrm{j}} + \mathrm{r}\sin\theta\underline{\mathrm{j}}$$
$$\underline{\mathrm{r}} = \mathrm{r}\left(\cos\theta\underline{\mathrm{j}} + \sin\theta\underline{\mathrm{j}}\right)$$
$$\underline{r} = r\underline{l}$$

$$\frac{d\underline{r}}{dt} = r\frac{d\underline{l}}{dt} + \frac{dr}{dt}\underline{l} = r\dot{l} + \dot{r}\underline{l} = r\underline{m}\dot{\theta} + \dot{r}\underline{l}$$
$$\underline{\dot{r}} = r\dot{\theta}\underline{m} + \dot{r}\underline{l}$$

$$\frac{d\dot{r}}{dt} = \dot{r}\dot{\theta}\underline{m} + r\ddot{\theta}\underline{m} + r\dot{\theta}\underline{\dot{m}} + \ddot{r}\underline{l} + \dot{r}\underline{\dot{l}} = \dot{r}\dot{\theta}\underline{m} + r\ddot{\theta}\underline{m} + r\dot{\theta}(-\dot{\theta}\underline{l}) + \ddot{r}\underline{l} + \dot{r}(\underline{m}\dot{\theta})$$
$$= (\ddot{r} - r\dot{\theta}^2)\underline{l} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{m}$$
$$\underline{\ddot{r}} = (\ddot{r} - r\dot{\theta}^2)\underline{l} + \frac{1}{r}(r^2\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{m} = (\ddot{r} - r\dot{\theta}^2)\underline{l} + \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})\underline{m}$$

4.2

Applying energy conservation law

$$\frac{1}{2}km\dot{r}^2 + \frac{1}{2}m(r^2\dot{\theta}^2 + \dot{r}^2) = \frac{1}{2}mu^2$$
$$(k+1)\dot{r}^2 + r^2\dot{\theta}^2 = u^2$$

(Please see the other two pages which attached separately for remain calculations. Remain of 4.2 and Q5)